Design and Analysis of Algorithms

Question #1:
You are given an array $A$ of $n$ distinct integers and another integer $x$. Give an $O(n \log n)$-time algorithm to determine whether or not there exists two elements in $A$ whose sum is exactly $x$.

Question #2:
VLSI databases commonly represent an integrated circuit as a list of rectangles. Assume that each rectangle is axis aligned (sides parallel to the $x$- and $y$-axes), so that we represent a rectangle by its minimum and maximum $x$- and $y$-coordinates. Give an $O(n \log n)$-time algorithm to decide whether or not a set of $n$ rectangles so represented contains two rectangles that overlap. Your algorithm need not report all intersecting pairs, but it must report that an overlap exists if one rectangle entirely covers another, even if the boundary lines do not intersect.

Question #3:
The transitive closure of a graph is a graph which contains an edge $(u, v)$ whenever there is a directed path from node $u$ to node $v$. Design an $O(VE)$-time algorithm for computing the transitive closure of a directed graph $G = (V, E)$.